Université Paris Saclay **MSc AI & MSc Data Science.**
TC2-Optimization Mid-Term Exam. Nov. 28th, 2024. Duration : 45'. Faïcel Chamroukhi

Instructions : : Only pens are allowed (documents and electronic devices are forbidden). It is forbidden to write with a pencil or a **red pen**. Your double exam sheet must include, in the designated area, your last name, first name, and signature. This designated area must be concealed by gluing. All your supplementary sheets must be numbered. The grading scale is provided for reference only.

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Exercice 1 *4pts (2 ; 2)*

1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function. What are the first order optimality conditions (FOC) of optimizing f without constraints
2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a twice-differentiable function. What are the second order optimality conditions (SOC) of optimizing f without constraints.

Exercice 2 *8pts (2 ; 3 ; 3)*

1. Let $f(x) = x^4 - 4x^2 + 1$. Is $f(x)$ convex ? Provide justification.
2. Let $f(x) = \max(x, 0)$. Is $f(x)$ convex ? Provide justification.
3. Show that the function $\sigma(x) = \frac{1}{1+e^{-x}}$ is convex when $x \leq 0$ by proving that its second derivative is non-negative.

Exercice 3 *8pts (4 ; 2 ; 2)* Consider the quadratic function $f(x) = \frac{1}{2}x^\top Ax + b^\top x + c$, where $A \in \mathbb{R}^{n\times n}$ is a symmetric matrix, x and $b \in \mathbb{R}^n$, and $c \in \mathbb{R}$.

1. Calculate the gradient and the Hessian of $f(x)$.
2. Discuss how the convexity properties of $f(x)$ change depending on A.
3. Given $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, determine if $f(x)$ is convex.