TD1 - November 2024.

Exercice 1. General questions about optimization.
Define the following terms:

(a) Objective function

(b) Feasible set

(c) Optimal solution

Exercice 2. Convexity

(a) Prove or disprove: The function $f(x) = |x|$ is convex.

(b) Consider the function $f(x) = x^2$ for $x \in \mathbb{R}$. Show that $f(x)$ is convex by using the second-derivative property.

(c) Let $f(x, y) = x^2 + y^2$. Determine if f is a convex function on $\mathbb{R}^2$.

Exercice 3. Differentiability

(a) Find the derivative of the function $f(x) = x^3$ at any point $x \in \mathbb{R}$.

(b) Determine if the function $f(x) = |x|$ is differentiable at $x = 0$. Justify your answer.

(c) Calculate the gradient of the function $f(x, y) = x^2 + y^2 + xy$ and determine if it is differentiable everywhere in $\mathbb{R}^2$.

Exercice 4. Optimality Conditions

(a) For the function $f(x) = x^2 + 4x + 4$, find the point where f achieves its minimum and verify that this point satisfies the second-order optimality condition.