

TD/TP - 2 - November 2024.

Exercise 1. Gradient Descent

1. Given $f(x) = x^2 + 4x + 4$, calculate the gradient of f .
2. Perform two steps of gradient descent starting from $x_0 = 2$ with a step size $\alpha = 0.1$.

Exercise 2. Towards Gradient Descent for Least Squares

1. For the least squares function $f(\mathbf{w}) = \frac{1}{2}\|y - X\mathbf{w}\|^2$, calculate the gradient of $f(\mathbf{w})$.
2. Calculate the Hessian of $f(\mathbf{w})$ and show that the problem is convex.

Exercise 3. Implementation of Gradient Descent

1. Implement in Python/Jupyter a simple gradient descent algorithm to minimize the function

$$f(x) = (x - 3)^2 + 4.$$

2. Plot the errors (in a log scale) to visualize the convergence rate. The goal is to visualize the path of gradient descent as it iteratively moves towards the minimum of $f(x)$.
3. Comment on your results.
4. Apply your algorithm to the function from Exercise 1.