TD: Gradient Descent for convex and smooth functions

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Convergence Analysis

We study the convergence for a fixed step size α. Prove the following result.

Theorem Assume that $f : \mathbb{R}^n \to \mathbb{R}$ is convex and L-smooth. If x^* is a critical point of f, i.e., $\nabla f(x^*) = 0$, then the the sequence $\{x^{(k)}\}$ generated by gradient descent

$$x^{(k+1)} = x^{(k)} + \alpha \nabla f(x^{(k)}),$$

with fixed step size $0 \leq \alpha \leq \frac{1}{L}$ satisfies:

$$f(x^{(k)}) - f(x^\star) \leq \frac{\|x^{(0)} - x^\star\|^2}{2\alpha k}.$$