Université Paris Saclay	**MSc AI & MSc Data Science.**
TC2-Optimization for Machine Learning	Faïcel Chamroukhi

Constrained Optimization
TD - 4 - December 5, 2024.

Exercice 1. Minimize:

$$f(x) = \sum_{i=1}^{n} x_i^2$$

Subject to:

$$\sum_{i=1}^{n} x_i = c, \quad x_i \geq 0 \quad \forall i = 1, \ldots, n,$$

where $c > 0$ is a constant.

Exercice 2. Consider the following problem of minimizing:

$$f(x) = x^\top Q x + c^\top x$$

subject to:

$$Ax = b, \quad x \geq \mathbf{0},$$

where:

- $x \in \mathbb{R}^n$ is the decision variable,
- $Q \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix,
- $c \in \mathbb{R}^n$ is a coefficient vector,
- $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$
- $x \geq \mathbf{0}$ ensures non-negativity of each component of x.

1. State the corresponding KKT conditions.
2. (optional, not treated in class) Solve the problem for $Q = 2I$, $c = (-4, -6)^\top$, $A = (1, 1)$, $b = 4$.

Exercice 3. (optional, not treated in class) Minimize:

$$f(x, y) = x^2 + y^2$$

Subject to:

$$x + y \leq 1, \quad x \geq 0, \quad y \geq 0$$